NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1557

COMPRESSIVE BUCKLING OF SIMPLY SUPPORTED PLATES WITH TRANSVERSE STIFFENERS

By Bernard Budiansky and Paul Seide

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Langley Field, Va.

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SUMMARY

Charts are presented for the analysis of the stability under longitudinal compression of simply supported rectangular plates with several equally spaced transverse stiffeners that have both torsional and flexural rigidity.

INTRODUCTION

The problem of determining the compressive buckling load of simply supported rectangular plates with identical equally spaced transverse stiffeners having no torsional rigidity has been treated by several writers. Timoshenko (reference 1), using the Rayleigh-Ritz energy method, obtained an approximate solution of this problem for plates with one stiffener and with three stiffeners. Lundquist (reference 2) reduced an infinite determinant obtained from the energy method and thereby derived an exact stability criterion in series form for plates having any finite number of stiffeners. In addition, Lundquist indicated a method involving graphical minimization for analyzing plates with infinitely many bays. In reference 3, Ratzersdorfer used a difference-equation approach to obtain a stability criterion in closed form (equivalent to Lundquist's series form) for plates with any finite number of stiffeners.

In the present paper the effect of stiffener torsional rigidity is taken into account. An appendix contains an exact Rayleigh-Ritz analysis of the compressive buckling of a simply supported plate that has, in addition to identical equally spaced intermediate stiffeners, end stiffeners of half the torsional rigidity of the intermediate stiffeners. (See fig. 1.) Stability criterions are obtained in series form and are then reduced to closed form to facilitate the subsequent derivation of limiting stability criterions for infinitely long plates with identical equally spaced stiffeners.

The theoretical results that are considered to be of most practical value are presented by means of charts giving nondimensional curves that may be used directly to analyze the stability of transversely stiffened plates. The curves were computed from the stability criterions for plates with infinitely many bays but are applicable, with small errors on the conservative side, to plates with four or more bays. The charts presented cover the practical range of stiffener spacings, between zero and one-half the plate width.

SYMBOLS

Plates:

x,y,z	coordinate axes (fig. 1)
w	displacement in z-direction
b	plate width
E	Young's modulus for plate
h	plate thickness
μ	Poisson's ratio for plate
D	plate flexural rigidity per unit width $\left(\frac{Eh^3}{12(1-\mu^2)}\right)$
L	distance between stiffeners
N	number of bays
β	aspect ratio of each bay $\left(\frac{L}{b}\right)$
E_rI_r	effective flexural rigidity of stiffener attached to plate
$\gamma = \frac{E_rI_r}{bD}$	
GJ	stiffener torsional rigidity
$\alpha = \frac{GJ}{bD}$	
N_x	critical compressive load per unit width

k	buckling load coefficient $\left(\frac{N_x b^2}{\pi^2 D}\right)$
P	total critical load $\left(N_x b\right)$
c	integer defining location of stiffener $\left(x_c = cL\right)$
q	integer defining buckling mode $(1 \leqq q \leqq n - 1)$
m,n,p,s	integers
δ_{mn}	Kronecker delta (1 if $m = n$; 0 if $m \neq n$)
A,A',B,B'	parameters defined in appendix

Columns:

P	column buckling load
L	length of each span
EI	column flexural stiffness
C	deflectional spring constant, force per unit deflection
K	rotational spring constant, moment per radian rotation

RESULTS AND DISCUSSION

Plates with Infinitely Many Bays

The theoretical stability criterions derived in the appendix for infinitely long simply supported plates with identical equally spaced transverse stiffeners relate three nondimensional parameters:

Buckling load parameter . . . . $\frac{PL^2}{bD}$

Stiffener flexural-rigidity parameter . . . . $\pi^4\left(\frac{L}{b}\right)^3\left(\frac{E_r I_r}{bD}\right)$

Stiffener torsional-rigidity parameter . . . . $\pi^2\left(\frac{L}{b}\right)\left(\frac{GJ}{bD}\right)$

Charts containing curves which show the relationship among three parameters are given in figures 2, 3, and 4 for plates having bays of aspect ratio L/b equal to 0.50, 0.35, and 0.20, respectively.

As the aspect ratio of the bay is decreased to zero, the curves of figures 2 to 4 converge to the curves of figure 5, taken from figure 5 of reference 4, for the buckling of an infinitely long column on equally spaced deflectional and rotational springs. The physical explanation of this convergence is that, as the stiffener spacing becomes smaller and smaller, the effect of transverse bending of the plate becomes less and less, until, in the limit, the plate buckles (except for the Poisson effect) like a column on elastic supports. The nondimensional parameters for the plate that are used were chosen so that, as L/b approaches zero, they become equivalent to the column parameters of reference 4, shown in figure 5. The curves of figure 4, for $\beta = 2$, and the column curves $(\beta = 0)$ of figure 5 are very close together, so that the column curves may be used to obtain good conservative approximations to the buckling loads of plates with bay aspect ratios less than 0.20. Figure 6, in which two of the curves of figure 4 are compared with the corresponding curves of figure 5, shows the error involved in the approximation to be small.

The data used in plotting the curves of figures 2 to 5 are presented in tables I to IV.

Plates with a Finite Number of Bays

In the appendix, exact stability criterions are given for the compressive buckling of a simply supported plate that has, in addition to any number of identical equally spaced intermediate stiffeners, end stiffeners of half the torsional rigidity of the intermediate stiffeners. The special end conditions were introduced to facilitate an exact solution by the Rayleigh-Ritz energy method.

Although the stability criterions could be used to construct charts similar to those presented for plates with infinitely many bays, such charts are not given because

(a) The charts for plates with infinitely many bays are suitable for the analysis of plates with four or more bays

(b) Charts for plates with one or two intermediate stiffeners would be of limited interest because of the special end conditions assumed in the analysis.

Plates with Four or More Bays

In figure 7, two of the curves of figure 4, for the plate with infinitely many bays, are compared with the corresponding curves for the plate with four bays, computed from the stability criterions for finite plates. This comparison indicates that a close approximation, on the conservative side, to the buckling load of plates with four or more bays may be obtained by means of the curves for infinitely many bays. The error involved, shown by figure 7 to be less than 10 percent for the four-bay case when $\beta = 0.20$, decreases as the number of bays increases.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., November 19, 1947

APPENDIX

DERIVATION OF THE STABILITY CRITERIONS

Plate with Finite Number of Bays

The method of solving for the stability criterions of simply supported plates having equally spaced transverse stiffeners is analagous to the method used in reference 4 to analyze the buckling of columns on equally spaced deflectional and rotational springs. A Fourier series is chosen to represent the deflection surface of the buckled plate, and the potential energy expression is minimized with respect to each of the unknown Fourier coefficients. The resulting equations are then separated into independent sets, each set containing the coefficients corresponding to a particular buckling mode. A general expression for the stability criterion for each buckling mode is derived.

Energy expressions.– The deflection surface of the buckled plate (see fig. 1) may be represented by the Fourier series

$$w = \sin\frac{\pi y}{b}\sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{NL} \tag{1}$$

When the initially straight stiffened plate buckles, the energy stored in it is

$$\begin{aligned} V &= \frac{D}{2}\int_0^{NL}\int_0^b \left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\mu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]\right\} dx\,dy \\ &= \frac{\pi^4}{8}NLbD\sum_{n=1}^{\infty} a_n^2\left(\frac{n^2}{N^2L^2} + \frac{1}{b^2}\right)^2 \end{aligned} \tag{2}$$

The bending energy stored in the stiffeners is

$$\begin{aligned} U_b &= \sum_{c=1}^{N-1}\frac{E_rI_r}{2}\int_0^b\left(\frac{\partial^2 w}{\partial y^2}\right)^2_{x=cL} dy \\ &= \frac{\pi^4}{4}\frac{E_rI_r}{b^3}\sum_{c=1}^{N-1}\left(\sum_{n=1}^{\infty} a_n \sin\frac{n\pi c}{N}\right)^2 \end{aligned} \tag{3}$$

The twisting energy of the stiffeners is

$$U_t = \frac{1}{2}\,\frac{GJ}{2}\int_0^b\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2_{x=0} dy + \sum_{c=1}^{N-1}\frac{GJ}{2}\int_0^b\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2_{x=cL} dy$$

$$+ \frac{1}{2}\,\frac{GJ}{2}\int_0^b\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)_{x=NL} dy$$

$$= \frac{\pi^4}{4}\,\frac{GJ}{N^2L^2b}\sum_{c=0}^{N}\left(\sum_{n=1}^{\infty} na_n \cos\frac{n\pi c}{N}\right)^2\left(\frac{1}{1 + \delta_{0c} + \delta_{Nc}}\right) \qquad (4)$$

The work done by the longitudinal load in shortening the plate is

$$W = \frac{N_x}{2}\int_0^{NL}\int_0^b\left(\frac{\partial w}{\partial x}\right)^2 dx\,dy$$

$$= \frac{\pi^2}{8}\,\frac{N_x b}{NL}\sum_{n=1}^{\infty} n^2 a_n^2 \qquad (5)$$

<u>Minimization</u>.— The buckling load may be found by minimizing the energy expression

$$F = V + U_b + U_t - W \qquad (6)$$

with respect to the a's. Substitution of equations (2), (3), (4), and (5) into equation (6) gives

$$F = \left\{\sum_{n=1}^{\infty} a_n^2\left[\left(n^2 + N^2\beta^2\right)^2 - kN^2\beta^2n^2\right] + 2N^3\beta^3\gamma\sum_{c=1}^{N-1}\left(\sum_{n=1}^{\infty} a_n \sin\frac{n\pi c}{N}\right)^2\right.$$

$$\left. + 2N\beta\alpha\sum_{c=0}^{N}\left(\sum_{n=1}^{\infty} na_n \cos\frac{n\pi c}{N}\right)^2\left(\frac{1}{1 + \delta_{0c} + \delta_{Nc}}\right)\right\}\frac{\pi^4 bD}{8(NL)^3} \qquad (7)$$

where

$$\beta = \frac{L}{b}$$

$$k = \frac{N_x b^2}{\pi^2 D}$$

$$\gamma = \frac{E_r I_r}{bD}$$

and

$$\alpha = \frac{GJ}{bD}$$

Then, minimizing F with respect to the a's yields

$$\begin{aligned}\frac{\partial F}{\partial a_n} &= 0\\ &= a_n\left[\left(n^2 + N^2\beta^2\right)^2 - kN^2\beta^2 n^2\right] + 2N^3\beta^3\gamma\sum_{m=1}^{\infty} a_m \sum_{c=1}^{N-1} \sin\frac{n\pi c}{N}\sin\frac{m\pi c}{N}\\ &\quad + 2N\beta\alpha n\sum_{m=1}^{\infty} m a_m \sum_{c=0}^{N}\cos\frac{n\pi c}{N}\cos\frac{m\pi c}{N}\left(\frac{1}{1 + \delta_{0c} + \delta_{Nc}}\right) \qquad (n=1,2,3,\ldots) \qquad (8)\end{aligned}$$

Stability criterions.– Equations (8) are similar to the corresponding equations (B8) of reference 4 and may be separated into $N + 1$ independent sets, each set corresponding to buckling in a particular mode. The application of the method of solution of reference 4 yields the stability criterions

$$\begin{aligned}&\left\{\frac{1}{\beta^3\gamma} - \sum_{s=0}^{\infty}\left[\frac{1}{R_{2s+\frac{q}{N}}} + \frac{1}{R_{2(s+1)-\frac{q}{N}}}\right]\right\}\left(\frac{1}{\beta\alpha} - \sum_{s=0}^{\infty}\left\{\frac{\left(2s + \frac{q}{N}\right)^2}{R_{2s+\frac{q}{N}}} + \frac{\left[2(s + 1) - \frac{q}{N}\right]^2}{R_{2(s+1)-\frac{q}{N}}}\right\}\right)\\ &\quad - \left\{\sum_{s=0}^{\infty}\left[\frac{2s + \frac{q}{N}}{R_{2s+\frac{q}{N}}} - \frac{2(s + 1) - \frac{q}{N}}{R_{2(s+1)-\frac{q}{N}}}\right]\right\}^2 = 0 \qquad \left(\frac{q}{N} = \frac{1}{N}, \frac{2}{N}, \ldots \frac{N-1}{N}\right) \qquad (9)\end{aligned}$$

where

$$R_{2s+\frac{q}{N}} = k\beta^2\left(2s + \frac{q}{N}\right)^2 - \left[\left(2s + \frac{q}{N}\right)^2 + \beta^2\right]^2$$

$$R_{2(s+1)-\frac{q}{N}} = k\beta^2\left[2(s + 1) - \frac{q}{N}\right]^2 - \left\{\left[2(s + 1) - \frac{q}{N}\right]^2 + \beta^2\right\}^2$$

and

$$\frac{1}{\beta\alpha} + \sum_{p=1,3,5,\ldots}^{\infty} \frac{2p^2}{\left(p^2 + \beta^2\right)^2 - k\beta^2p^2} = 0 \qquad (10)$$

corresponding to buckling of the plate with nodes at the stiffeners and with a symmetrical buckling configuration in each bay, and

$$\frac{1}{\beta\alpha} + \sum_{p=2,4,6,\ldots}^{\infty} \frac{2p^2}{\left(p^2 + \beta^2\right)^2 - k\beta^2p^2} = 0 \qquad (11)$$

corresponding to buckling of the plate with nodes at the stiffeners and with an antisymmetrical buckling configuration in each bay. Equations (9), (10), and (11) reduce to those obtained in reference 2 if the stiffener torsional rigidity is zero.

The buckling criterion of equation (9), for a particular value of q, corresponds to a buckling configuration having a Fourier expression that contains only the coefficients

$$a_q,\ a_{2N+q},\ a_{4N+q},\ \ldots$$

and

$$a_{2N-q},\ a_{4N-q},\ \ldots$$

The criterion will be satisfied by many different buckling loads for given values of β, γ, and α, each of which will correspond to a buckling configuration in which one of the Fourier coefficients given previously is dominant. For practical stiffener spacings, the lowest buckling load generally corresponds to a mode with a_q dominant, and hence with q buckles in the longitudinal direction. It is possible, however, that for large stiffener spacings $(\beta > \sqrt{2})$, for which the natural half wave length of the unstiffened buckled plate is less than the stiffener spacing, the lowest buckling load corresponds to a mode in which a coefficient other than a_q is dominant.

Closed-form solutions.- The infinite series in equations (9), (10), and (11) may be evaluated, as in reference 4, by expressing the general term of each series as the sum of partial fractions and recombining these fractions to obtain other series readily evaluated by equations 6.495-1 and 6.495-2 of reference 5. The closed-form stability criterions obtained in this manner are

$$\left[\frac{1}{\gamma} - \frac{\pi}{4\sqrt{k-4}}\left(\frac{B \sin A'}{\cos\frac{\pi q}{N} - \cos A'} - \frac{A \sin B'}{\cos\frac{\pi q}{N} - \cos B'}\right)\right]\left[\frac{1}{\alpha} - \frac{\pi}{4\sqrt{k-4}}\left(\frac{A \sin A'}{\cos\frac{\pi q}{N} - \cos A'} - \frac{B \sin B'}{\cos\frac{\pi q}{N} - \cos B'}\right)\right]$$

$$- \left[\frac{\pi \sin\frac{\pi q}{N}}{2\sqrt{k(k-4)}} \frac{\cos A' - \cos B'}{\left(\cos\frac{\pi q}{N} - \cos A'\right)\left(\cos\frac{\pi q}{N} - \cos B'\right)}\right]^2 = 0 \qquad \left(\frac{q}{N} = \frac{1}{N}, \frac{2}{N}, \ldots \frac{N-1}{N}\right) \tag{12}$$

and

$$\frac{1}{\alpha} + \frac{\pi}{4\sqrt{k-4}}\left(A \tan\frac{A'}{2} - B \tan\frac{B'}{2}\right) = 0 \tag{13}$$

and

$$\frac{1}{\alpha} - \frac{\pi}{4\sqrt{k - 4}}\left(A \cot \frac{A'}{2} - B \cot \frac{B'}{2}\right) = 0 \qquad (14)$$

where

$$A = 1 + \sqrt{1 - \frac{4}{k}}$$

$$B = 1 - \sqrt{1 - \frac{4}{k}}$$

$$A' = \frac{\pi\beta\sqrt{k}}{2}\left(1 + \sqrt{1 - \frac{4}{k}}\right)$$

$$B' = \frac{\pi\beta\sqrt{k}}{2}\left(1 - \sqrt{1 - \frac{4}{k}}\right)$$

Equations (12), (13), and (14) constitute the complete set of closed-form stability criterions. The correct criterion for any given values of γ and α is that which yields the lowest buckling-load coefficient. The equations reduce to those obtained by Ratzersdorfer (reference 3) if the stiffener torsional rigidity is equal to zero.

Plate with Infinitely Many Bays

The stability equation for a plate with infinitely many bays is obtained by expanding equation (13) and minimizing the result with respect to q/N; this procedure gives

$$\cos \frac{\pi q}{N} = \frac{\cos A' + \cos B'}{2} + \frac{\pi\gamma}{8\sqrt{k - 4}}(B \sin A' - A \sin B')$$

$$+ \frac{\pi\alpha}{8\sqrt{k - 4}}(A \sin A' - B \sin B') \qquad \left(0 < \frac{q}{N} < 1\right) \quad (15)$$

Substitution of equation (15) into equation (12) yields

$$
\begin{aligned}
&\pi^2\gamma^2(B \sin A' - A \sin B')^2 + 8\pi\gamma\sqrt{k-4}\,(\cos A' - \cos B')(B \sin A' + A \sin B') \\
&\quad - 2\pi^2\gamma\alpha\left[AB(\cos A' - \cos B')^2 + 2AB(1 - \cos A' \cos B') - \left(A^2 + B^2\right) \sin A' \sin B'\right] \\
&\quad + 8\pi\alpha\sqrt{k-4}\,(\cos A' - \cos B')(A \sin A' + B \sin B') \\
&\quad + \pi^2\alpha^2(A \sin A' - B \sin B')^2 + 16(k-4)(\cos A' - \cos B')^2 = 0 \qquad (16)
\end{aligned}
$$

which is the stability criterion for a plate with infinitely many bays when $0 < \frac{q}{N} < 1$.

When q/N is equal to 1, equation (12) yields two independent criterions:

$$
\alpha = -\frac{4\sqrt{k-4}}{\pi}\left(\frac{1}{A \tan \frac{A'}{2} - B \tan \frac{B'}{2}}\right) \qquad (17)
$$

corresponding to buckling with no bending of the ribs and with a symmetrical buckling configuration in each bay, and

$$
\gamma = -\frac{4\sqrt{k-4}}{\pi}\left(\frac{1}{B \tan \frac{A'}{2} - A \tan \frac{B'}{2}}\right) \qquad (18)
$$

corresponding to antisymmetrical buckling with no rib twisting.

When q/N is equal to 0, equation (12) yields two other independent criterions:

$$\alpha = \frac{4\sqrt{k - 4}}{\pi}\left(\frac{1}{A \cot \frac{A'}{2} - B \cot \frac{B'}{2}}\right) \tag{19}$$

corresponding to antisymmetrical buckling with no bending of the ribs, and

$$\gamma = \frac{4\sqrt{k - 4}}{\pi}\left(\frac{1}{B \cot \frac{A'}{2} - A \cot \frac{B'}{2}}\right) \tag{20}$$

corresponding to symmetrical buckling with no rib twisting.

The curves of figures 2 to 4 were plotted by means of equations (16), (17), and (18) in a manner similar to that outlined in appendix C of reference 4. Equations (19) and (20) were not used since, for the values of β considered in this paper, they yield higher buckling loads than equations (17) and (18).

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TABLE I

DATA FOR FIGURE 2

[$\beta = 0.50$]

$\frac{PL^2}{bD}$ \ $\pi^2\left(\frac{L}{b}\right)\left(\frac{GJ}{bD}\right)$	$\pi^4\left(\frac{L}{b}\right)^3\left(\frac{E_rI_r}{bD}\right)$ = 0	2	5	10	20	50	∞
9.870	0						
11.550	4.95	0					
13.070	10.85	4.11	0				
14.170	17.32	8.03	3.26	0			
15.056	25.29	11.34	5.94	2.48	0		
15.417	33.44	12.92	7.24	3.73	1.33	0	
15.421	35.11	12.96	7.25	3.74	1.34	.01	0
15.567		13.55	7.72	4.18	1.82	.71	.71
17.392		22.96	14.71	11.11	9.80	9.80	9.80
19.038		43.23	21.92	18.33	17.99	17.99	17.99
21.463			34.27	30.17	30.17	30.17	30.17
23.297			55.10	39.37	39.37	39.37	39.37
25.903				52.84	52.84	52.84	52.84
28.190				72.55	64.66	64.66	64.66
33.551					93.21	93.21	93.21
33.702					95.57	93.75	93.75
39.373						125.01	125.01
39.399						125.26	125.09
44.875							156.58

TABLE II

DATA FOR FIGURE 3

[$\beta = 0.35$]

$\frac{PL^2}{bD}$ \ $\pi^2\left(\frac{L}{b}\right)\left(\frac{GJ}{bD}\right)$	$\pi^4\left(\frac{L}{b}\right)^3\left(\frac{E_rI_r}{bD}\right)$ = 0	2	5	10	20	50	∞
4.836	0						
6.512	2.75	0					
8.234	7.18	2.62	0				
9.685	12.40	6.07	2.32	0			
10.997	19.16	10.16	5.44	2.24	0		
12.020	27.51	14.03	8.46	4.84	2.18	0	
12.426	38.39	15.68	9.76	6.04	3.34	1.29	0
13.601		21.11	13.76	9.70	7.08	5.75	5.75
15.657		36.08	22.04	17.57	16.00	16.00	16.00
16.025		46.66	23.66	19.09	17.81	17.81	17.81
20.310			57.97	39.53	39.46	39.46	39.46
25.231				74.34	64.62	64.62	64.62
30.406					92.40	92.40	92.40
30.701					97.20	94.24	94.24
36.382						124.92	124.92
36.457						125.66	125.08
42.007							157.69

NACA

TABLE III

DATA FOR FIGURE 4

$[\beta = 0.20]$

$\frac{PL^2}{bD}$ \ $\pi^2\left(\frac{L}{b}\right)\left(\frac{GJ}{bD}\right)$	$\pi^4\left(\frac{L}{b}\right)^3\left(\frac{E_rI_r}{bD}\right)$ = 0	2	5	10	20	50	∞
1.579	0						
3.274	1.39	0					
5.001	4.35	1.48	0				
6.708	8.86	4.44	1.68	0			
8.261	14.81	8.19	4.26	1.68	0		
9.658	22.88	12.60	7.57	4.22	1.79	0	
10.675	39.26	16.45	10.62	6.72	3.98	1.83	0
12.140		23.21	15.31	11.14	8.46	7.22	7.22
14.252		43.55	24.04	18.77	17.77	17.77	17.77
14.324		47.31	24.38	19.59	18.13	18.13	18.13
18.567			58.41	39.60	39.54	39.54	39.54
23.493				74.75	64.90	64.90	64.90
28.583					91.89	91.89	91.89
29.019					97.36	94.03	94.03
34.706						124.97	124.97
34.796						125.70	125.26
40.275							157.78

TABLE IV

DATA FOR FIGURE 5

$\frac{PL^2}{EI}$ \ $\frac{KL}{EI}$	$\frac{CL^3}{EI}$ = 0	2	5	10	20	50	∞
0	0						
1.708	.73	0					
3.467	3.04	.68	0				
5.218	6.97	3.09	.87	0			
6.906	12.58	6.72	3.09	.88	0		
8.468	20.36	11.28	6.56	3.31	1.30	0	
9.870	39.48	16.50	10.50	6.71	3.94	1.88	0
11.370		23.39	15.25	10.79	7.73	7.41	7.41
13.461		45.35	24.24	19.48	17.86	17.86	17.86
13.491		47.43	24.33	20.06	17.99	17.99	17.99
17.749			58.71	39.57	39.26	39.26	39.26
22.667				74.95	64.80	64.80	64.80
27.952					92.71	92.71	92.71
28.164					97.43	93.86	93.86
33.907						125.87	125.87
33.931						126.15	126.05
39.478							157.91

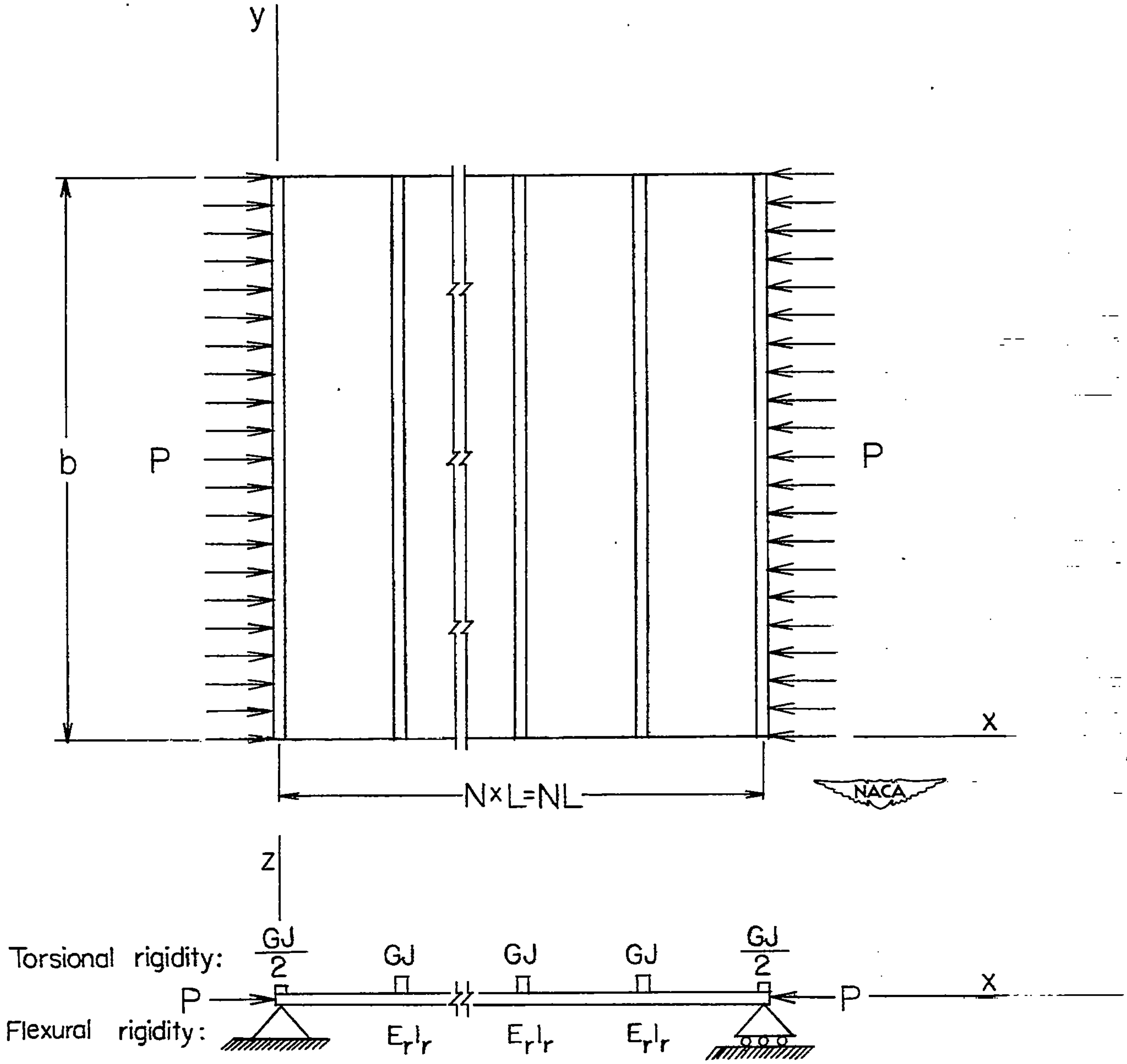

Figure 1.- Plate with transverse stiffeners.

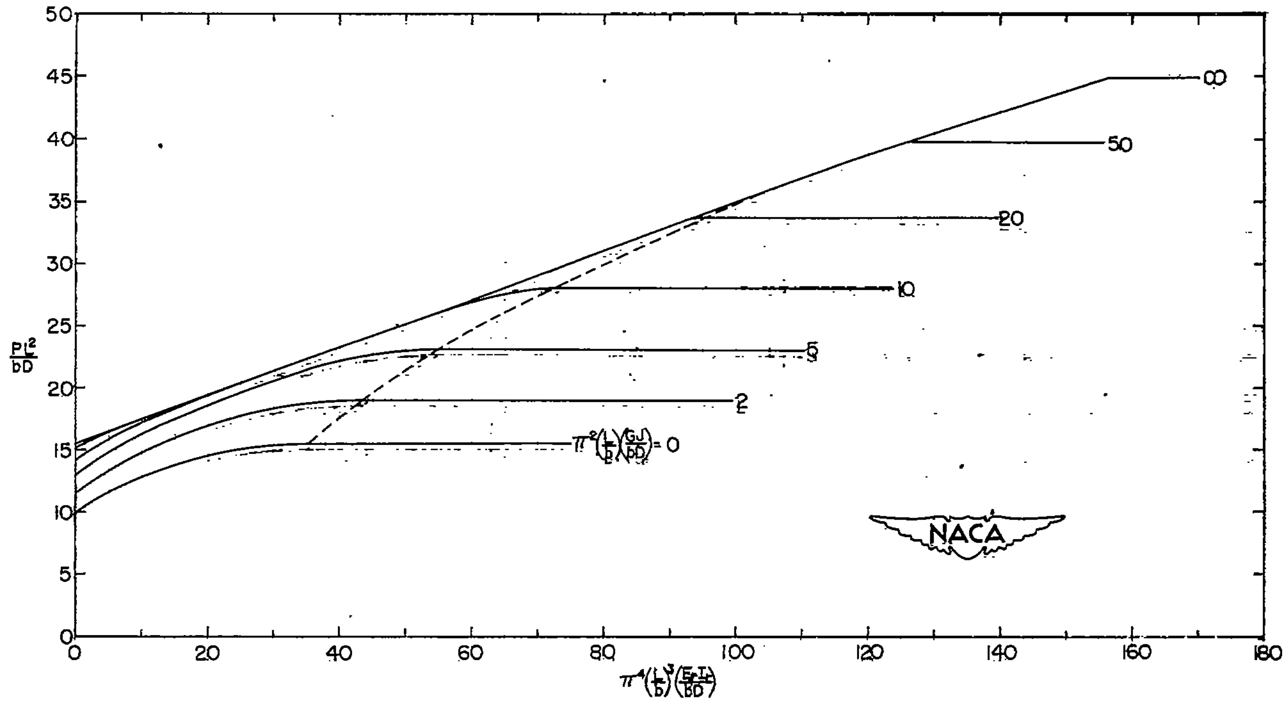

Figure 2.- Buckling curves for plate with infinitely many bays. $\beta = 0.50$.

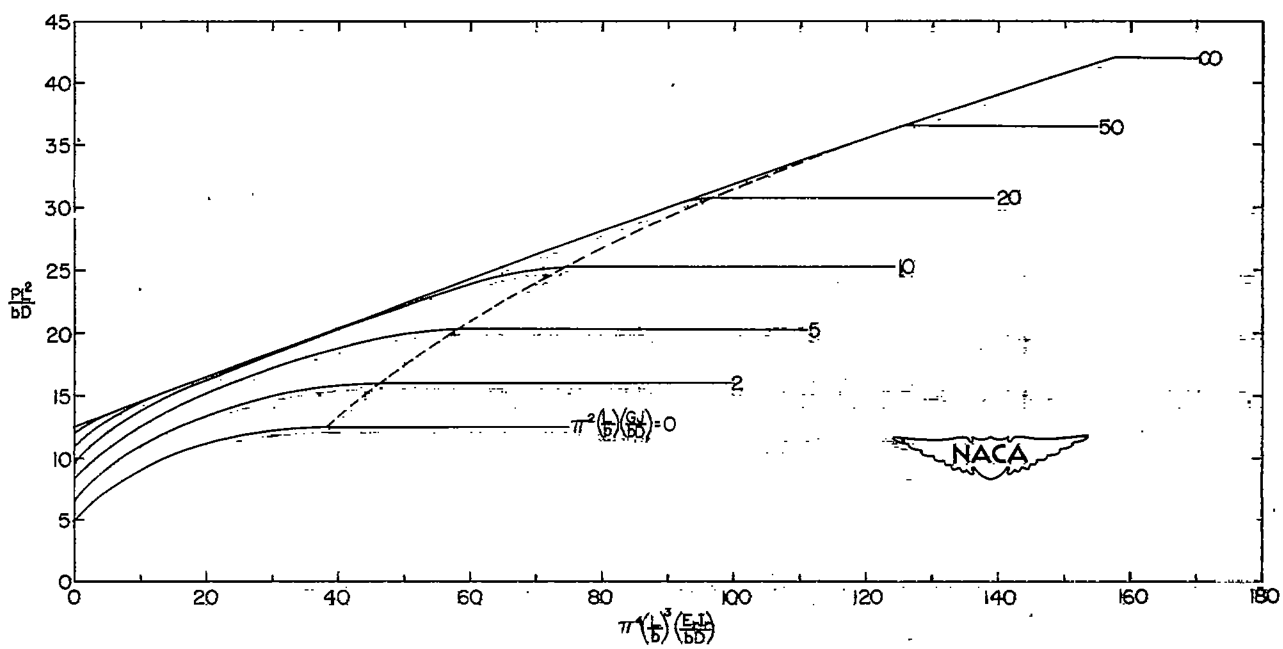

Figure 3.- Buckling curves for plate with infinitely many bays. $\beta = 0.35$.

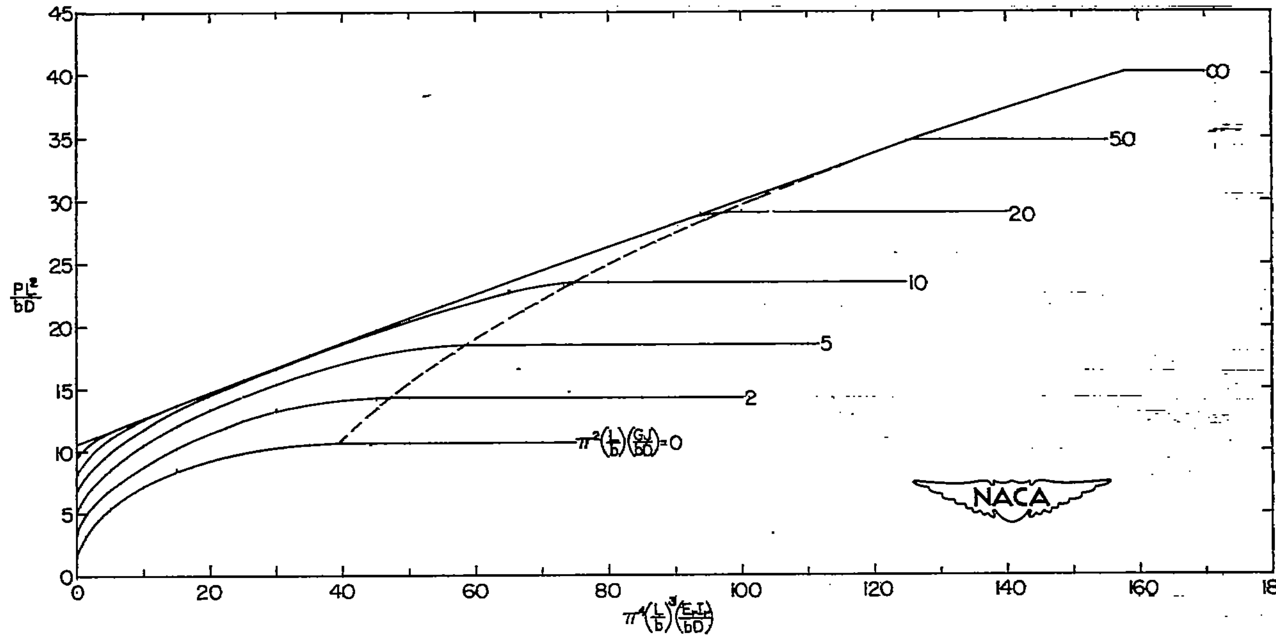

Figure 4.- Buckling curves for plate with infinitely many bays. β=0.20.

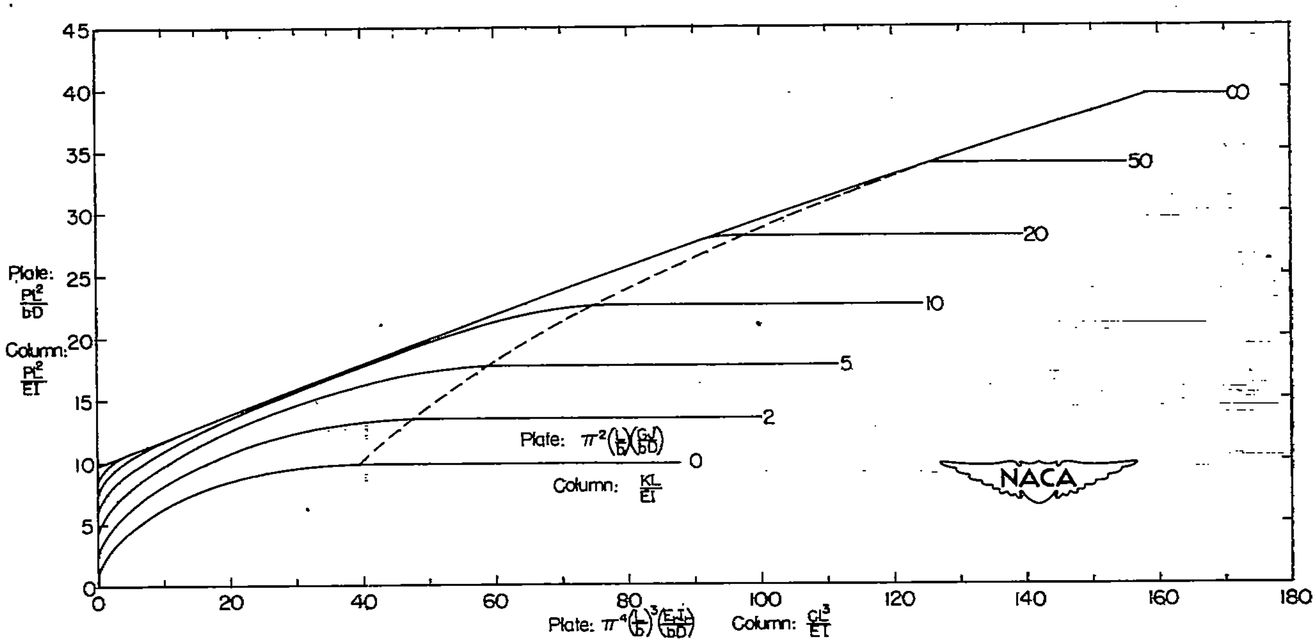

Figure 5.- Approximate buckling curves for plate with infinitely many bays. β<0.20.
(Buckling curves for column with infinitely many spans).

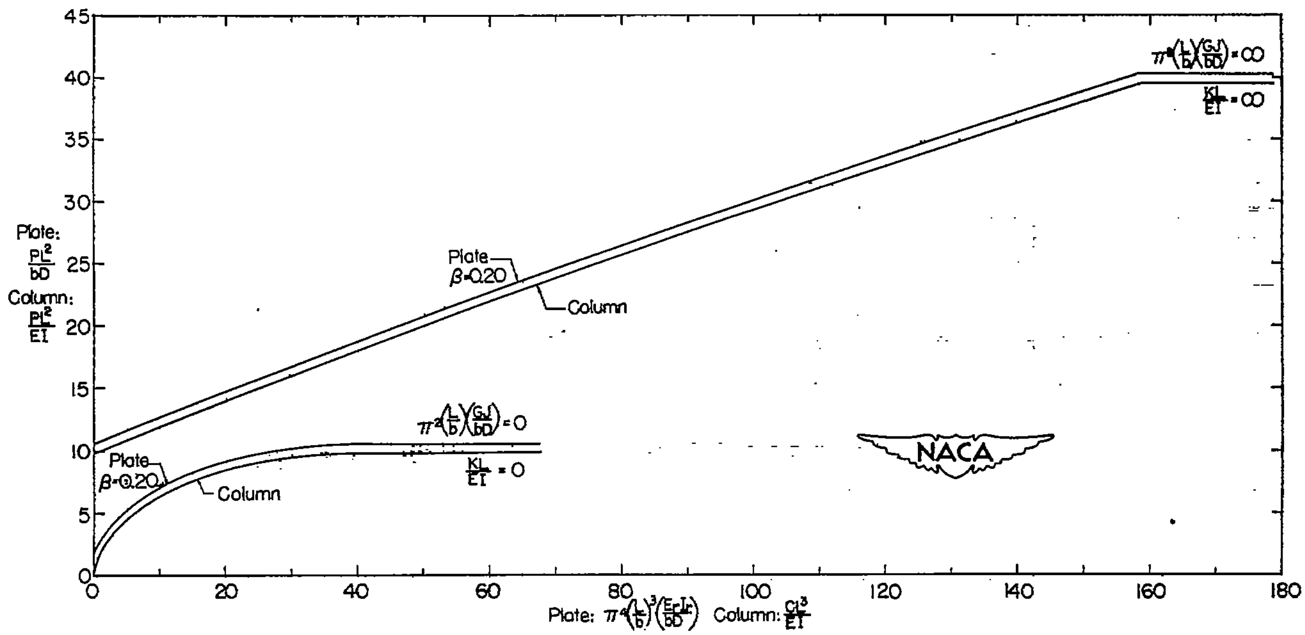

Figure 6.- Comparison of buckling curves for stiffened plate (β=0.20) and elastically supported column.

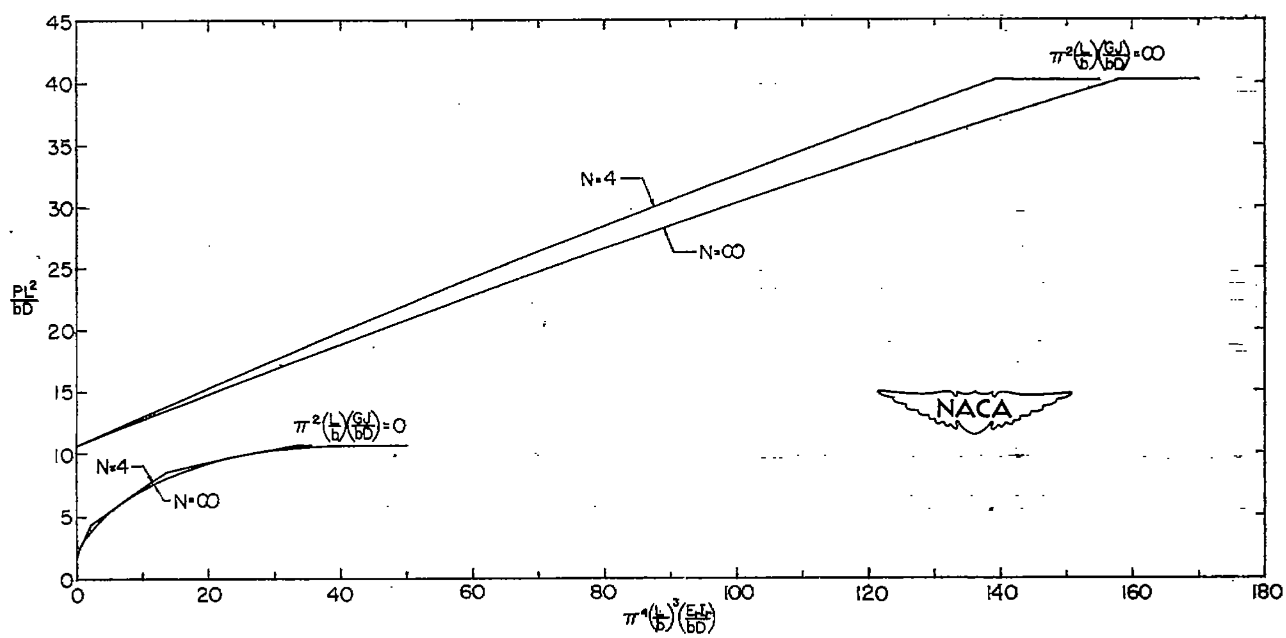

Figure 7.- Comparison of buckling curves for plates with four bays and infinitely many bays. β= 0.20.